Uplands Junior School

Uplands Junior School Calculation Policy 2020

CPA and Bar Model approach

Progression towards a standard written method of calculation

INTRODUCTION

This calculation policy has been written in line with the programmes of study taken from the revised National Curriculum for Mathematics (2014). It provides guidance on appropriate calculation methods and progression. The content is set out in yearly blocks under the following headings: addition, subtraction, multiplication and division. This policy supports the White Rose maths scheme used throughout the school.

AIMS OF THE POLICY

- To ensure consistency and progression in our approach to calculation
- To ensure that children can use methods accurately with confidence and understanding
- To ensure that children develop an efficient, reliable, formal written method of calculation for all operations

CPA APPROACH

Concrete, pictorial, abstract (CPA) concepts should not be confused as differentiation for lower, middle, higher attaining children. CPA is an approach to be used with the whole class and teachers should promote each area as equally valid. Manipulatives in particular must not be presented as a resource to support the less confident or lower attaining pupils.

Concrete representation— a pupil is first introduced to an idea or skill by acting it out with real objects. This is a 'hands on' component using real objects and is a foundation for conceptual understanding.
Pictorial representation – a pupil has sufficiently understood the 'hands on' experiences performed and can now relate them to representations , such as a diagram or picture of the problem.
Abstract representation—a pupil is now capable of representing problems by using mathematical notation, for example 12 x 2 = 24.

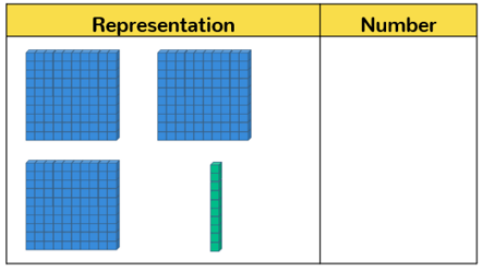
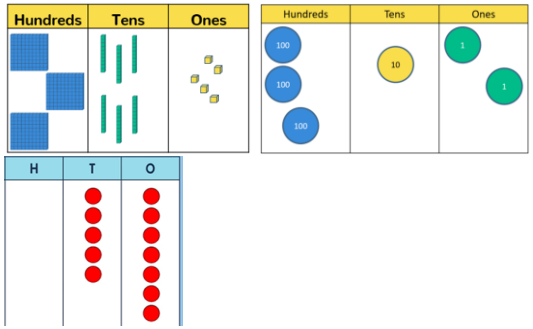
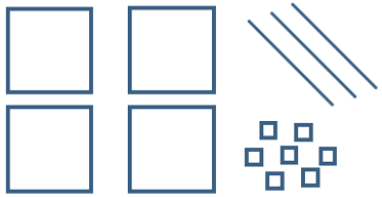
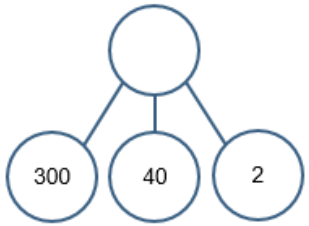
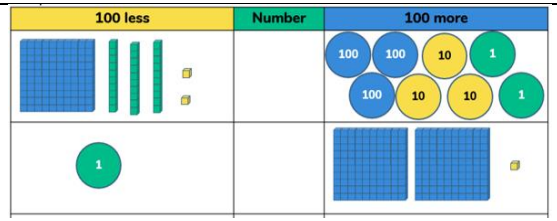
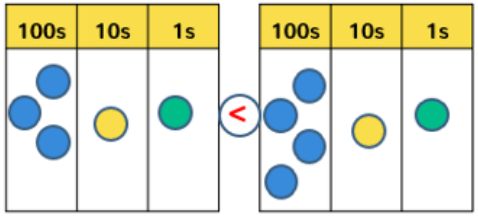
- Children need to use concrete resources before they progress to pictorial and abstract representations. This CPA (concrete, pictorial and abstract) approach needs to be available to children throughout school, as and when necessary. Use of manipulatives (numicon, base 10 apparatus/dienes, HTO counters etc.) helps reinforce understanding and provides support when calculating mentally, mentally with jottings, using expanded methods and formal written methods. Use of the bar model, number lines and part-part whole diagrams are recommended.

- Children should progress between the stages working towards formal written methods (where appropriate), once they have mastered each stage. However, they should not be hurried and, after the method has been taught, children should still be able to make their preferred choice of the most appropriate, efficient and accurate method for them. Previous stages may need to be revisited to consolidate understanding when introducing a new strategy.

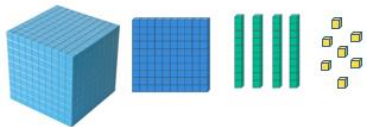
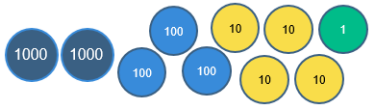
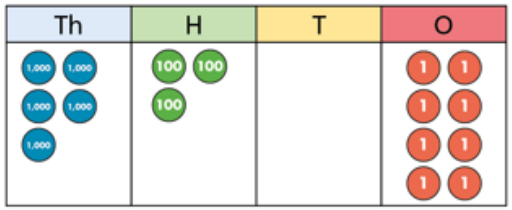
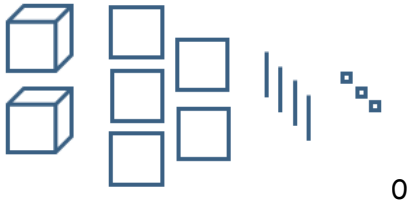
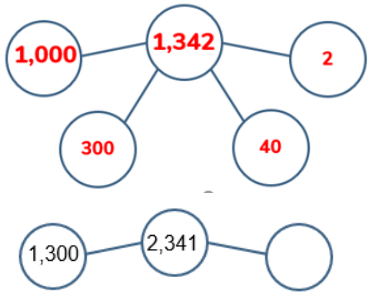
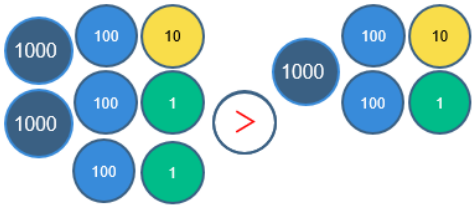
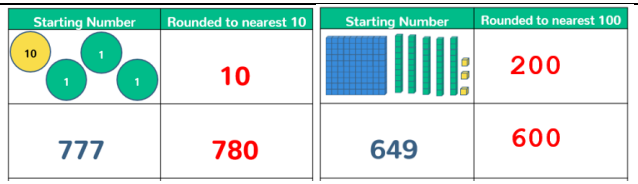
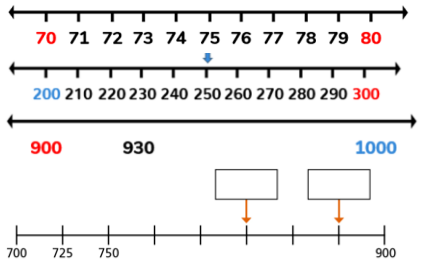
- As new methods of calculations are introduced, children should have the opportunity to examine them, alongside the method they have consolidated, to make connections between the methods and establish the similarities and differences between them.

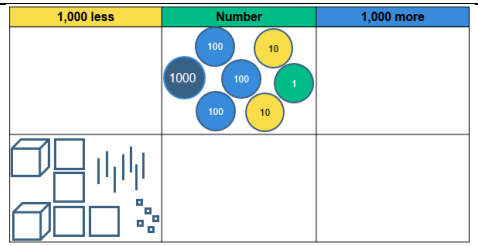
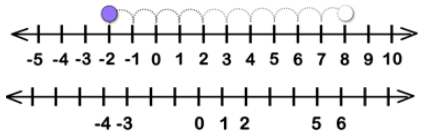
Place Value

Year 3 Place Value

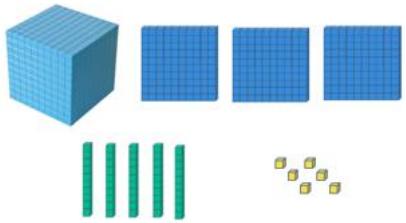
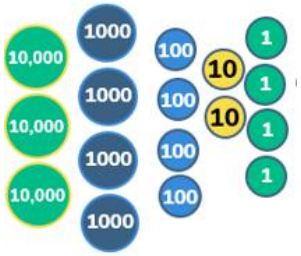
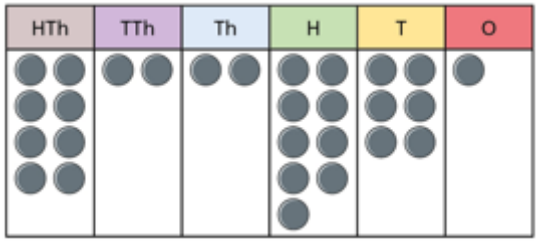
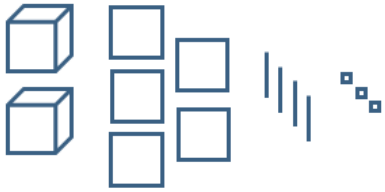
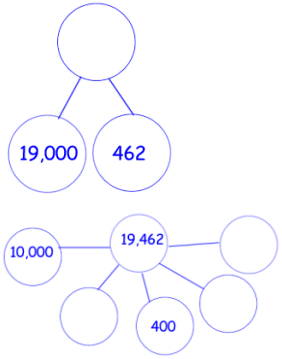
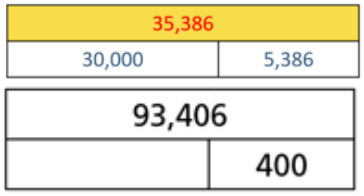
Objective	Concrete Resources	Pictorial/Bar models	Abstract
Pictorial and abstract representations should be used alongside the concrete at every step			
• Identify, represent and estimate numbers using different representations. • Read and write numbers in numerals and in words. • Recognise the place value of each digit in numbers https://www.youtube.com/watch?v=vBIZal-8Kr4 https://www.youtube.com/watch?v=NNf4qXqTv7I	**Base 10**  **Place value counters/Double sided counters** **Place value charts** 	Drawings to represent base 10  Number lines Part- whole model 	Numbers written in digits Writing numbers in words **and** numerals
• Find 10, 100, 1000 more or less than a given number. https://www.youtube.com/watch?v=gqUtj9rkYCU https://www.youtube.com/watch?v=a9j9JJ6LADY			Written calculations ___ + 100 = Number track 150 | 200 | | 300 | 350 |
• Compare and order numbers		Use a mixture of concrete, pictorial and abstract – can children identify numbers? Can they compare and order between both	Ordering numbers in digits only. Use of < and > symbols

Year 4 Place Value

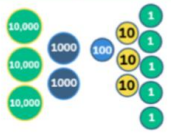
Objective	Concrete Resources	Pictorial/Bar models	Abstract
Pictorial and abstract representations should be used alongside the concrete at every step			
• Identify, represent and estimate numbers using different representations. • Recognise the place value of each digit in a **four-digit** number. Partition numbers. (thousands, hundreds, tens and ones) https://www.youtube.com/watch?v=vBIZal-8Kr4 https://www.youtube.com/watch?v=NNf4qXqTv7I	**Base 10 apparatus** **Place value counters** **Place value grids**    Th H T O	Drawings to represent base 1  0 **Part-whole model**  1,000 1,342 2 300 40 1,300 2,341	Numbers written in digits Writing numbers in words and numerals 5,308 = ☐ + ☐ + ☐
• Order and compare numbers beyond 1000	Use resources as above 	Use a mixture of concrete, pictorial and abstract – can children identify numbers? Can they compare and order between both?	Ordering numbers in digits only. Use of < and > symbols 6,000 ◯ 3,981
• Round any number to the nearest 10, 100 or 1000	 Starting Number / Rounded to nearest 10: 10 — 777 / 780 Starting Number / Rounded to nearest 100: 200 — 649 / 600	**Number lines**  70 71 72 73 74 75 76 77 78 79 80 200 210 220 230 240 250 260 270 280 290 300 900 930 1000 700 725 750 900	Round given numbers in digits Number / Rounded to the nearest 10 / Rounded to the nearest 100 / Rounded to the nearest 1,000 755 2,904 5,997

• Count in multiples of 6, 7, 9. 25 and 1000. • Find 1000 more or less than a given number. https://www.youtube.com/watch?v=bG1B5d5GMHM https://www.youtube.com/watch?v=XlIkwIfV_XQ			Number tracks 200 225 250 ___ ___ ___ ___ ___ 750 725 700 ___ ___ ___ ___ ___ ___ 3,000 ___ ___ 6,000 7,000 Written calculations ___ + 100 =
• Count backwards through zero to include negative numbers.			Number tracks -7 -6 ___ ___ ___ ___ ___ ___

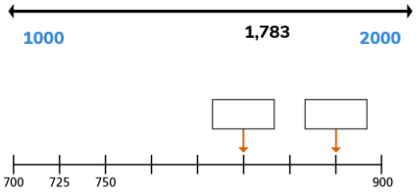
Year 5 Place Value

Objective	Concrete Resources	Pictorial/Bar models	Abstract
Pictorial and abstract representations should be used alongside the concrete at every step			
• Read and write numbers to at least 1000000 and determine the value of each digit.	Base 10 apparatus – **up to 10,000 only**  **Place value counters** **Place value grids**  	Drawings to represent base 10 – **up to 10,000 only**  **Part-whole model**  **Bar model** 	Numbers written in digits Writing numbers in words and numerals _____ = 50,000 + 8,000 + 790
• Order and compare numbers to at least 1000000	**As above** 	Use a mixture of concrete, pictorial and abstract – can children identify numbers? Can they compare and order between both?	Ordering numbers in digits only. Use of < and > symbols 4,000 + 10,000 ◯ 13,000 19,000 + 70 + 200 ◯ 19,270

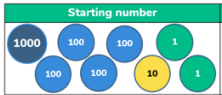
- Round any number up to 1000000 to the nearest 10, 100, 1000, 10000 and 100000

Starting Number	Rounded to nearest 1,000
	
34,983	

Number lines



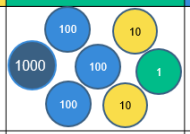
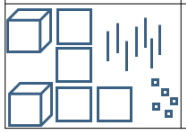
Round given numbers in digits

Starting number	Nearest 10	Nearest 100	Nearest 1,000
			
			

Start Number	Nearest 10	Nearest 100	Nearest 1,000
254			
2,349			
	5,880		

- Count forwards or backwards in steps of powers of 10 for any given number up to 1000000.

https://www.youtube.com/watch?v=bG1B5d5GMHM
https://www.youtube.com/watch?v=XlIkwIfV_XQ

1,000 less	Number	1,000 more
		
		

Number tracks

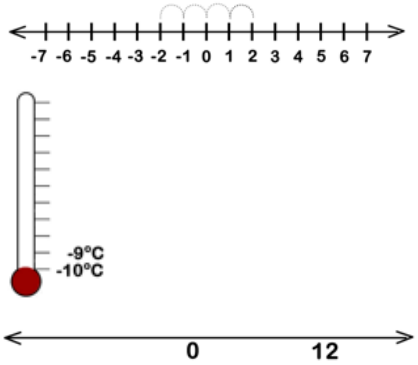
6,300		6,280	6,270			

1,650, 2,650, ____, ____, 5,650

Number	10 more	100 more	1,000 more	10,000 more	100,000 more
25					
250					
2,500					
25,000					
250,000					

- Interpret negative numbers in context, count forwards and backwards with positive and negative whole numbers including through zero.

Thermometer

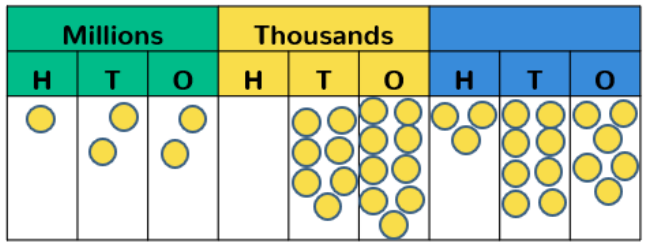
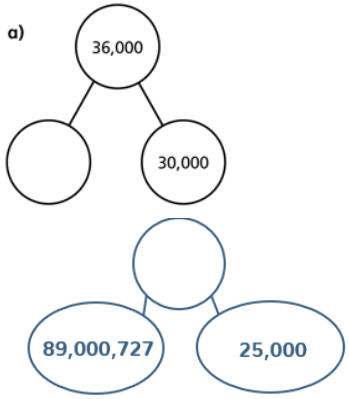
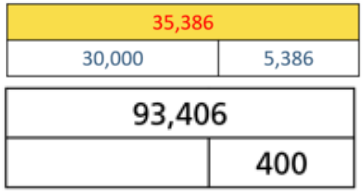
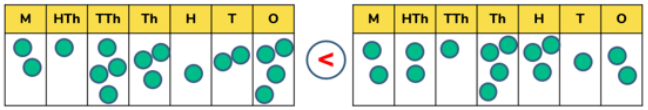


Find the difference between numbers – positive and negative.

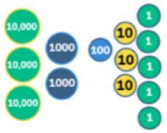
Identify numbers between given points

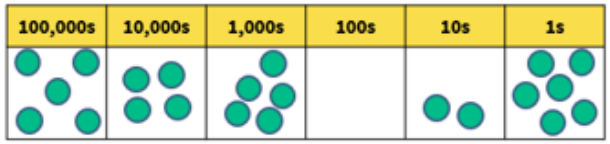
Calculate temperature rise and fall.

Year 6 Place Value

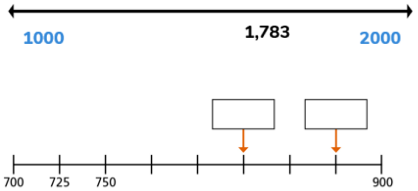
Objective	Concrete Resources	Pictorial/Bar models	Abstract
Pictorial and abstract representations should be used alongside the concrete at every step			
• Read and write numbers to at least 10,000,000 and determine the value of each digit.	**Place value counters** **Place value grids** 	**Part-whole model**  **Bar model** 	Numbers written in digits Writing numbers in words and numerals _____ = 50,000 + 8,000 + 790 103,531,052 = 100,000,000 + _____ + 500,000 + _____ + 1,000 + ___ + 2
• Order and compare numbers to at least 10,000,000	**As above**  2,143,124 < 2,214,312	Use a mixture of concrete, pictorial and abstract – can children identify numbers? Can they compare and order between both?	Ordering numbers in digits only. Use of < and > symbols 4,000 + 10,000 ◯ 13,000 19,000 + 70 + 200 ◯ 19,270

- Round any whole number to a required degree of accuracy.

Starting Number	Rounded to nearest 1,000
	
34,983	



Number lines

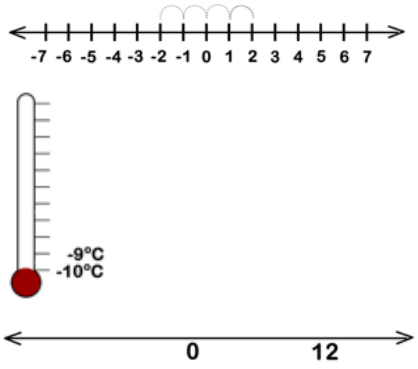


Round given numbers in digits

Rounded to the nearest	147,283	68,547	1,656,908	900,571
10				
100				
1,000				
10,000				
100,000				

- Use negative numbers in context, and calculate intervals across zero.

Thermometer



11, 1, __, -19, -29, __

−3 + 5 =

1 − 4 =

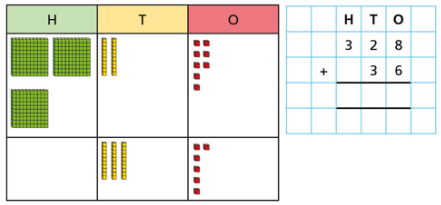
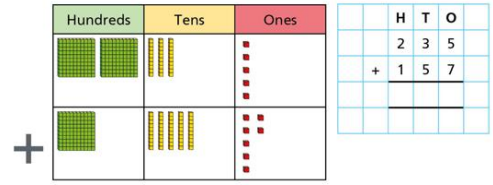
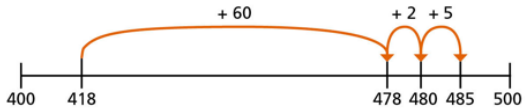
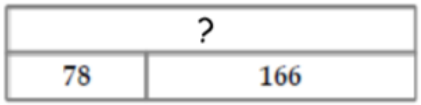
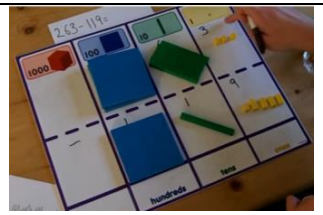
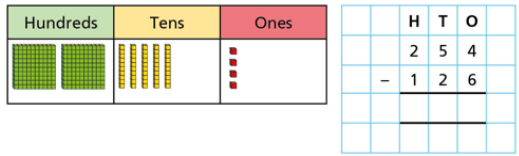
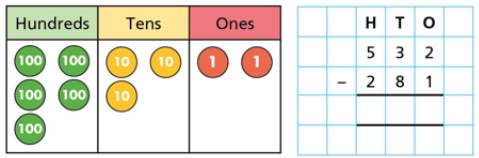
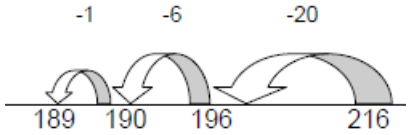
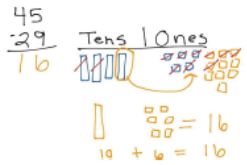
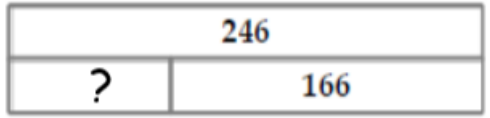
−4 + 4 =

7 more than -3 =

Calculate temperature rise and fall.

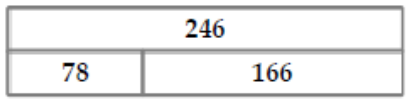
Addition and Subtraction

Year 3 Addition and Subtraction

Objective	Concrete Resources	Pictorial/Bar models	Abstract
Pictorial and abstract representations should be used alongside the concrete at every step			
Add numbers mentally, including: a three-digit number and ones; a three-digit number and tens; a three digit number and hundreds. Add numbers with up to three digits, using formal written methods of columnar addition. https://www.youtube.com/watch?v=hHM25Nx4vhq https://www.youtube.com/watch?v=38dc-eS8S_k https://www.youtube.com/watch?v=PRAOFeuaaVU	 **Base 10 apparatus** **Place Value counters/charts**  	**Draw counters and charts**  **Number lines**  **Bar models**  **78 + 166 = ___** **see below**	*(Partitioning)* 200 + 60 + 3 + 100 + 10 + 9 300 + 70 + 12 300 + 80 + 2 *(Column method)* 2 6 3 + 1 1 9 3 8 2 1 H T O 1 0 9 + 2 5 5
Subtract numbers mentally, including: a three-digit number and ones; a three-digit number and tens; a three digit number and hundreds. Subtract numbers with up to three digits, using formal written methods of columnar subtraction. https://www.youtube.com/watch?v=pADFYrGdyYE https://www.youtube.com/watch?v=-JUxKogWhF8 https://www.youtube.com/watch?v=sTILCPp6q2c	 **Base 10 apparatus** **Place Value counters/charts**  	Develop the use of empty number line with calculations that bridge 100:   **Bar models** 	*(Partitioning)* 50 13 200 + ~~60~~ + ~~3~~ - 100 + 10 + 9 100 + 40 + 4 *(Column method)* 5 1 2 ~~6~~ 3 - 1 1 9 1 4 4 H T O 2 0 - 1 8 2 9

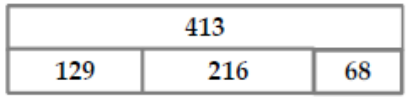
Solve problems, including missing number problems, using number facts, place value, and more complex addition and subtraction.

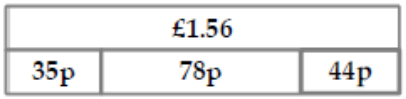
Bar Modelling approach:

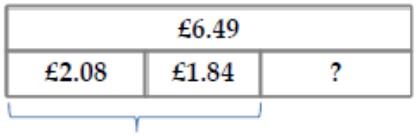


78 + 166 = 246 166 + 78 = 246

246 – 78 = 166 246 – 166 = 78



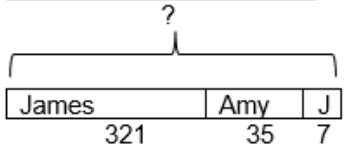




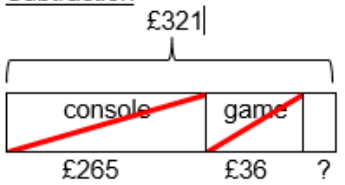
£2.08 + £1.84 = £3.92

£6.49 - £3.92 = £2.57

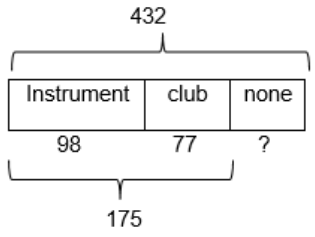
Addition of up to 3 numbers



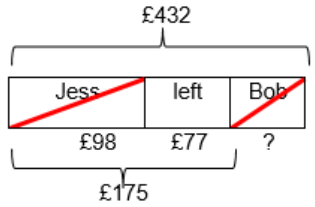
Subtraction



Missing numbers



or

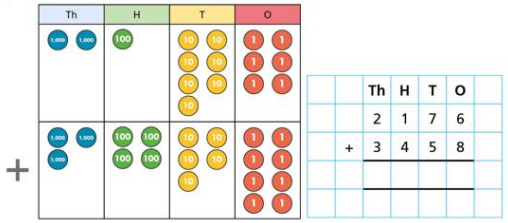
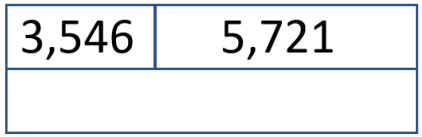
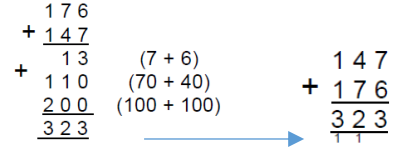
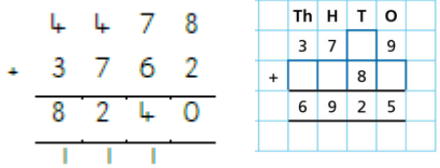
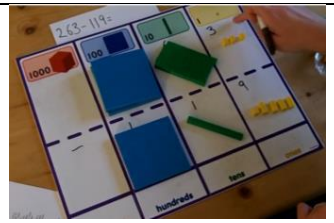
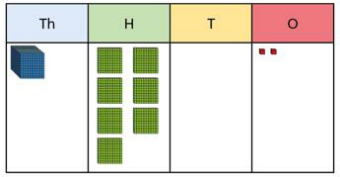
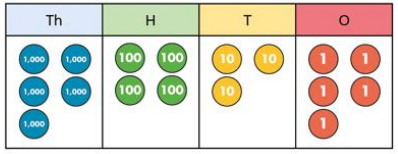
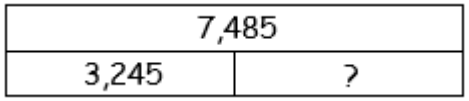
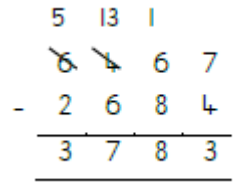
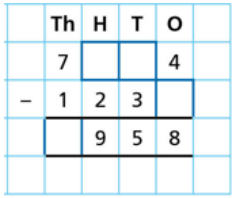


Children explore relationships within addition and subtraction to help when solving inverse operation problems, and to check answers to column addition and subtraction questions. They also continue to use bar models to show addition of three numbers, and more complex subtraction. Children begin to use bar models to represent more complex word problems. They begin with recapping simple, one-step problems before moving onto two-step problems, using their model to decide which operation to use. Other concepts, such as money, should also be linked. They should become comfortable with explaining how the model represents the problem, and should be able to think of their own word problems based on a given bar model.

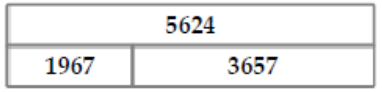
Year 3 Problems

- Show the bar model to represent 139 + 282
- Show the bar model to represent 319 – 148
- Work out the missing answer in this bar model. Write down 4 number sentences that are shown by the model
- Jen takes 38 pens out of a packet. There were 100 to start with. How many are left in the packet?
- Bill has £1.46 on one piggy bank, £2.39 in another and £1.27 in a third. How much money does he have altogether? Show this on a bar model.
- Katie wants a new game that costs £6.49. Her mum gives her £2.08 and her dad gives her £1.84. How much more does she need? Show this on a bar model.
- There are 334 children at Springfield School and 75 at Holy Trinity Nursery. How many children are there altogether?
- Gemma collected 293 badges but she gave 45 of them to her friend, Rebecca. How many badges did she have left?
- Seven people each put five pens into a pot. Carmen then takes out fifteen pens. How many pens are left?
- If you spend 61p at the corner shop, how much change do you get from £1.00?

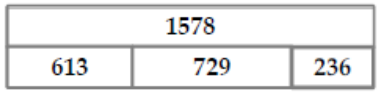
Year 4 Addition and Subtraction

Objective	Concrete Resources	Pictorial/Bar models	Abstract
Pictorial and abstract representations should be used alongside the concrete at every step			
Add numbers with up to 4 digits using a formal written method. Estimate and use inverse operations to check answers to calculations. https://www.youtube.com/watch?v=hHM25Nx4vhq https://www.youtube.com/watch?v=38dc-eS8S_k https://www.youtube.com/watch?v=PRAOFeuaaVU https://www.youtube.com/watch?v=uLyLYqXXywA	 **Base 10 apparatus** **Place value counters/charts** 	Continue to teach the use of empty number lines for 3 or 4 digit numbers as appropriate (see Year 3 guidance) **Bar models**  3,546 + 5,721 = 5,721 + 3,546 =	Continue to develop the formal method of addition, with 3 to 4-digit numbers, by revisiting the expanded method, if necessary  
Subtract numbers with up to 4 digits using a formal written method. Estimate and use inverse operations to check answers to calculations. https://www.youtube.com/watch?v=pADFYrGdyYE https://www.youtube.com/watch?v=-JUxKoqWhF8 https://www.youtube.com/watch?v=sTILCPp6q2c	 **Base 10 apparatus** **Place value counters/charts**  Use the place value chart to complete the subtractions. 	Continue to teach the use of empty number lines for 3 or 4 digit numbers as appropriate (see Year 3 guidance) **Bar models**  Identify all known facts – use inverse operations. 7,485 – 3,245 = ? 7,485 - ? = 3,245 3,245 + ? = 7,485 ? + 3,245 = 7,485	Continue to develop the formal method of subtraction by revisiting the expanded method, if necessary  Use the language of place value to ensure understanding. In this example we have **exchanged** one hundred for 10 tens. 

Solve addition and subtraction two-step problems in context, deciding on which operation and method to use and why.	**Bar Modelling approach:** Continue as methods in Year 3 with numbers up to 4 digits.

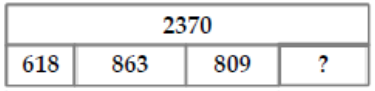


1967 + 3657 = 5624 3657 + 1967 = 5624
5624 – 1967 = 3657 5624 – 3657 = 1967

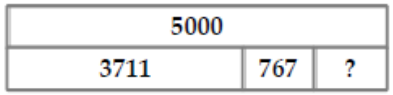


1578 – 613 – 729 = 236

A postman needs to deliver 2370 letters. He delivers 618 on Monday, 863 on Tuesday and 809 on Wednesday. How many does he have left to deliver?



The bar model should help children to see that they should add 618, 863 and 809. Once they have done this, the familiarity of the model will help them to see that they should take away the total from 2370.

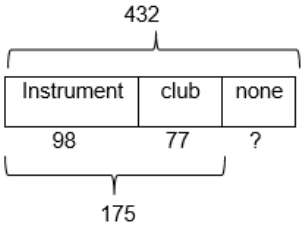


4478

3711 + 767 = 4478

5000 – 4478 = 522

Missing numbers

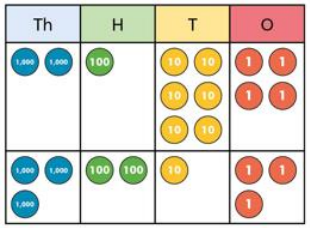
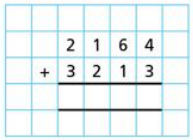
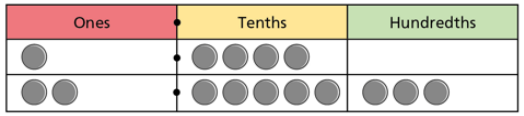
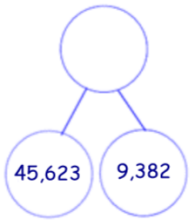
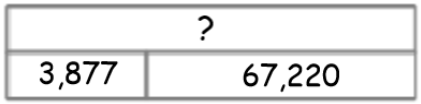
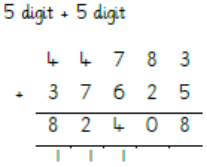
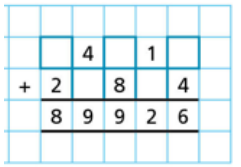
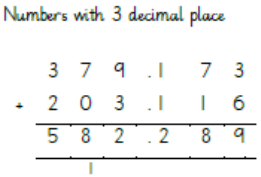
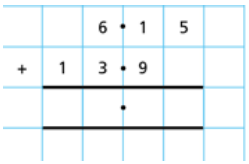
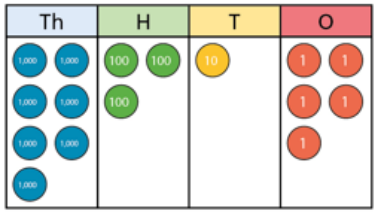
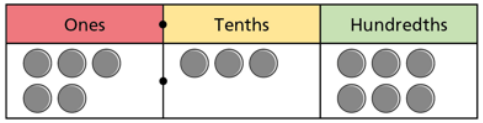
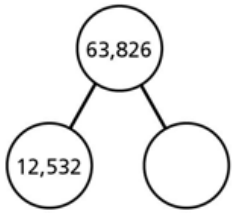
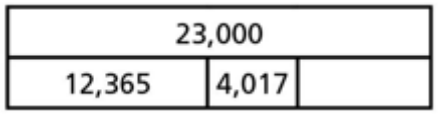
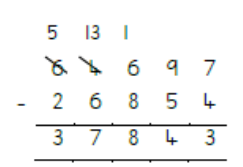
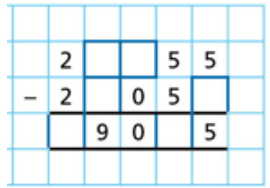
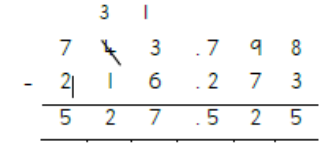
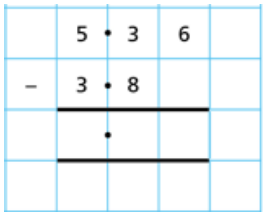


Year 4 Problems

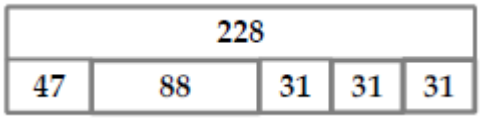
- Show the bar model to represent 2787 + 2009 + 1829
- Show the bar model to represent 7262 – 2786
- Work out the missing answer in *this* bar model. Write down 4 number sentences that are shown by the model.
- There are 2,131 books in the library. Year 2 borrow 117 books and Year 3 borrow 89 books. How many books will be left when Year 4 arrive in the library?
- There are 3,711 people sat in a concert hall. 767 people have already left the concert. The hall can seat 5,000 people altogether. How many seats were empty for the concert?
- Martin has saved £6.78 and spends £4.69. How much does he have left?

Years 5 and 6 Addition and Subtraction

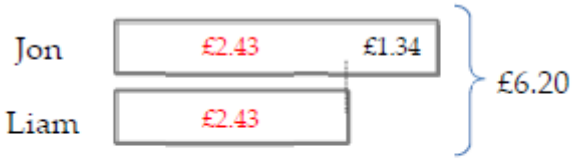
Year 6: Focus on **application of abstract methods** to solve problems – use of concrete and pictorial representations to support where needed. Extend to 6-digit numbers.

Objective	Concrete Resources	Pictorial/Bar models	Abstract
	Pictorial and abstract representations should be used alongside the concrete at every step		
Add whole numbers and decimals with more than 4 digits, including using formal written methods (columnar addition and subtraction)	**Base 10 apparatus** **Place value counters/charts**   	 **Part/Whole models** **Bar models** 	5 digit + 5 digit   Numbers with 3 decimal place  
Subtract whole numbers and decimals with more than 4 digits, including using formal written methods (columnar addition and subtraction)	**Base 10 apparatus** **Place value counters/charts**  	 **Part/Whole models** **Bar models** 	5 digit - 5 digit   Numbers with 3 decimal place  

Solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why.	**Bar Modelling approach:** **Children move on to problems requiring them to calculate with more than 3 numbers. Bar modelling involving addition and subtraction in Y5/6 is likely to also require some modelling of multiplicative and divisive reasoning and fractions, decimals and percentages reasoning. Children begin to combine the 4 operations more within multi-step word problem, using bar models to structure their thoughts and decide on appropriate calculations.**



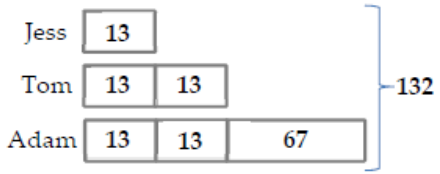
On Monday, Gita reads 47 pages of her book. She reads 88 pages the next day. If the book has 228 pages, and she splits the remaining pages between the next 3 days, how many pages does she read on these days?



£6.20 – £1.34 = £4.86

£4.86 ÷ 2 = £2.43

Eg. Jon has £1.34 more than Liam. Altogether, they have £6.20. How much do they each have?



132 – 67 = 65

65 ÷ 5 = 13

Jess: 13
Tom: 26
Adam: 93

Year 5/6 Problems

- Sam has half the amount of money Lara has. Emma has twice as much as Lara. If they have £1.61 altogether, how much do they each have?
- Tom has twice as many colouring pencils as Jess, but 67 less than Adam. They have 132 altogether. How many do they each have? (see example above)
- Lara delivers a total of 567 letters. She delivered twice as many letters on Tuesday as she did on Monday. On Wednesday, she delivered 32 more than on Tuesday. How many did she deliver each day?
- Harry had £137 in his money box. He spent £65 on some computer games, and then shared what was left between himself and his 2 brothers. How much did each brother get?